Lorentz Anomaly and 1+1-Dimensional Radiating Black Holes[1]

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ABSTRACT

The radiation from the black holes of a 1+1-dimensional chiral quantum gravity model is studied. Most notably, a non-trivial dependence on a renormalization parameter that characterizes the anomaly relations is uncovered in an improved semiclassical approximation scheme; this dependence is not present in the naive semiclassical approximation.

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The realization that the study of some 1+1 dimensional quantum gravity theories, like the CGHS model[1], might give the possibility to explore black hole quantum mechanics in a rather simple (compared to the physical 3+1 dimensional context) setting, has led to great excitement. (See Ref.[2] for a review of related results.) In particular, there have been several attempts[3] to modify the CGHS model in a way that would allow a consistent analysis (from beginning to end) of evaporating black holes. These attempts have been only partly successful[3], but there might still be room for improvement since just a limited class of modifications have been considered. Other recent papers have been devoted to the investigation of the role of anomalies in the quantum mechanics of 1+1 dimensional black holes. Most notably, again in studies of the CGHS model, it has been found[4, 5] that the semiclassical (quantized matter fields in a background geometry) evaluation of the Hawking radiation[6] is essentially insensitive to the addition of the local counterterm that converts the Weyl anomaly into an anomaly for diffeomorphisms of nonconstant Jacobian[5, 7, 8].

In this Letter, we consider a modification of the CGHS model in which chiral fermion fields replace the usual boson fields as the matter content, and (accordingly) the geometry is described by the *zweibein* e^a_μ instead of the metric $g_{\mu\nu}$. ($g_{\mu\nu} \equiv e^a_\mu e^b_\nu \eta_{ab}$ and $\eta \equiv diag(-1, 1)$.) We study the radiation of the black holes of this model in the semiclassical approximation, with particular attention to the role played by the anomalies[2].

Chiral quantum gravity theories in 1+1-dimensions have a rich anomaly structure, and no inconsistency has been encountered in their previous studies (see, for example, Ref.[9, 10]); it is therefore quite natural to use one such theory for the investigation of the role of anomalies in the quantum mechanics of 1+1-dimensional black holes. We also hope that the analysis here presented will motivate future studies in which the features of some chiral quantum gravity theories be exploited in obtaining a fully consistent description of black hole evaporation.

The model we consider has action $S_D + S^+_m + S^-_m$, where

$$S_D = \int d^2x \, E \, e^{-2\phi} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] \, , \tag{1}$$

$$S^\pm_m = -\frac{1}{2} \sum_{n=1}^{N_\pm} \int d^2x \; E \; e^\mu_a \left(\bar{\psi}^{(\pm)}_n \gamma^a \frac{1 \pm i\gamma_5}{2} \partial_\mu \psi^{(\pm)}_n - \partial_\mu \bar{\psi}^{(\pm)}_n \gamma^a \frac{1 \pm i\gamma_5}{2} \psi^{(\pm)}_n \right) . \tag{2}$$

e^a_μ, ϕ, and $\psi^\pm_n$ are the *zweibein*, dilaton, and matter fields respectively. E denotes the determinant of the *zweibein* (which equals $\sqrt{-det(g)}$). The curvature R can be written in terms of the spin-connection $\omega_\mu = \epsilon^{\alpha\beta} e^a_\mu \eta_{ab} \partial_\alpha e^b_\beta / E$ (we choose to work without torsion), as $R = 2\epsilon^{\mu\nu}\partial_\mu\omega_\nu/E$. Note that S_D,$S^\pm_m$ are invariant under (tangent space) Lorentz transformations and diffeomorphisms. $S^\pm_m$ are also invariant under Weyl transformations.

With a straightforward generalization of the corresponding analysis[2] of the CGHS model, it is easy to verify that the classical equations of motion of the model have black hole solutions. In particular, the gravitational collapse of the left-moving shock wave considered in Refs.[1, 2, 5] forms the same classical black hole that it forms in the CGHS model. We shall limit our analysis to this example of black hole, so that our results can be more easily compared to those of [1, 2, 5]; moreover, like in Refs.[1, 2, 5], we introduce light-cone coordinates and work in conformal gauge. [Note that, since ours is a *zweibein theory*, the conformal gauge is characterized by $e^\pm_\mu = e^\rho \delta^\pm_\mu$; however, this implies that $g_{+-} = -e^{2\rho}/2$, $g_{++} = g_{--} = 0$.]

[2]We refer the reader to Ref.[12] for a discussion of certain other aspects (different from the ones we are here concerned with) of the physics of black holes in presence of chiral matter; note, however, that in Ref.[12] at the quantum level only the special case with equal number of left-moving fields and right-moving fields was considered and therefore the structure of the anomalies was much simpler.

For an observer $\hat{e}^a_\mu(\sigma)$ that is *zweibein minkowskian* ($\hat{e}^a_\mu = \delta^a_\mu$) at past null infinity[5], the solution of the classical equations of motion that corresponds to the black hole considered in Refs.[1, 2, 5] has conformal factor[3]

$$\hat{\rho} = -\frac{1}{2} \ln \left[1 + \Theta(\sigma^+ - \sigma^+_0) \frac{a}{\lambda} e^{\lambda \sigma^-} \left(e^{\lambda(\sigma^+_0 - \sigma^+)} - 1 \right) \right] \ . \tag{3}$$

In the semiclassical analysis of Hawking radiation one evaluates the quantum matter energy-momentum tensor in the background geometry of the black hole. In our *zweibein* model the matter energy-momentum tensor is

$$\hat{T}^\mu_a \equiv - < \frac{1}{E} \frac{\delta [S^+_m + S^-_m]}{\delta e^a_\mu} >_{\hat{e}} \ \ , \tag{4}$$

and for the example of black hole background that we are considering the most interesting physical information is encoded[2] in the value of $\hat{T}_{--} \equiv \hat{T}^\mu_a \hat{e}^a_- \hat{g}_{\mu -}$ on[4] $\mathcal{I}^+_R$.
From the results of Ref.[11] it follows that the diffeomorphism invariant quantization of our chiral fields in a background geometry $\hat{e}^a_\mu$ leads to the anomaly relations (N.B.: $\hat{X} \equiv X(\hat{e})$)

$$\hat{\nabla}_\mu \ \hat{T}^\mu_a \ = \ -\frac{\hat{e}^\mu_a \hat{\omega}_\mu}{24\pi} \left(\frac{cq}{2} \hat{R} - 2h \hat{\nabla}_\mu \hat{\omega}^\mu \right) \ , \tag{5}$$

$$\hat{e}^a_\mu \ \hat{T}^\mu_a \ = \ \frac{1}{24\pi} \left[(c+h) \hat{R} + cq \hat{\nabla}_\mu \hat{\omega}^\mu \right] \ , \tag{6}$$

$$\epsilon^a{}_b \ \hat{e}^b_\mu \ \hat{T}^\mu_a \ = \ \frac{1}{24\pi} \left(\frac{cq}{2} \hat{R} - 2h \hat{\nabla}_\mu \hat{\omega}^\mu \right) \ , \tag{7}$$

where $c \equiv (N_+ + N_-)/2$, $q \equiv (N_+ - N_-)/(N_+ + N_-)$, and h is a free[5] renormalization parameter (the coefficient of a local counterterm). The emergence of the free parameter h in chiral quantum gravity theories should be understood in complete analogy with the emergence of the Jackiw-Rajaraman parameter in the quantization of the matter fields of the Chiral Schwinger model[14].

For arbitrary c,q,h the right-hand sides of Eqs.(5), (6), (7) do not transform covariantly under Lorentz and Weyl transformations, also $\hat{T}^\mu_a$ does not transform covariantly under these transformations. Instead, in spite of the unusual aspect of Eq.(5) (that is due to the Lorentz anomaly), $\hat{T}^\mu_a$ transforms covariantly under diffeomorphisms. For $h = q = 0$ (diffeomorphism and Lorentz invariant Dirac fermions) the anomaly relations (5), (6), (7) are equivalent to the ones encountered[1, 2] in the CGHS model, and $\hat{T}^\mu_a$ transforms covariantly under both Lorentz transformations and diffeomorphisms. Also special is the case $q = 0$ (Dirac fermions), $h = -c$ which leads to a $\hat{T}^\mu_a$ that is traceless and covariant under diffeomorphism transformations, but Lorentz anomalous.

The Eqs.(5), (6), (7) can be used to evaluate $\hat{T}_{--}$ on I^+_R. Indeed, these anomaly relations completely determine, modulo the imposition of physical boundary conditions at past null

[3]Like in Refs.[2, 5], a denotes the magnitude of the shock wave, and the shock wave moves along $\sigma^+ = \sigma^+_0$.

[4]Like in Ref.[2, 5], $\mathcal{I}^+_R$ ($\mathcal{I}^-_R$) is the future (past) null infinity for right-moving light rays, and analogously $\mathcal{I}^+_L$ ($\mathcal{I}^-_L$) is the future (past) null infinity for left-moving light rays.

[5]Obviously, here "free" should be understood in the usual field theoretical sense, *i.e.* one can choose freely among the values of h consistent with unitarity (for example in ordinary theories one cannot choose a negative mass parameter). In Ref.[9] it was argued that only $h > 0$ would lead to a unitary chiral quantum gravity theory; we shall come back to this later.

infinity, the four components of $\hat{T}^{\mu}_{a}$. This is analogous to the ordinary CGHS case, the only difference being that here we have one more component of the energy-momentum tensor (the antisymmetric component coming from the Lorentz anomaly) and one more anomaly relation (the one expressing the Lorentz anomaly).

Imposing as physical boundary condition[2, 5] that $\hat{T}^{\mu}_{a}$ vanish on $\mathcal{I}^{-}_{L}$ (*i.e.* $\sigma^{+}=-\infty$), and that there be no incoming radiation along $\mathcal{I}^{-}_{R}$ (*i.e.* $\sigma^{-}=-\infty$) except for the classical shock-wave at $\sigma^{+}=\sigma^{+}_{0}$, with a straightforward (but lengthy) derivation one finds that

$$\hat{T}_{--}=\frac{1}{12\pi}\left[\left(c+h-\frac{cq}{2}\right)\left(\partial^{2}_{\sigma^{-}}\hat{\rho}-(\partial_{\sigma^{-}}\hat{\rho})^{2}\right)-\frac{cq}{2}(\partial_{\sigma^{-}}\hat{\rho})^{2}\right]\ , \tag{8}$$

The physical interpretation of the energy flux seen on $\mathcal{I}^{+}_{R}$ is clearest[2] to observers that are asymptotically minkowskian on $\mathcal{I}^{+}_{R}$, but from Eq.(3) one sees that the $\hat{e}^{a}_{\mu}(\sigma)$ observer is not minkowskian on $\mathcal{I}^{+}_{R}$ (whereas it is minkowskian on $\mathcal{I}^{-}_{R,L}$). An observer $\check{e}^{a}_{\mu}(y)$ that is *zweibein minkowskian* on $\mathcal{I}^{+}_{R}$ can be obtained from $\hat{e}^{a}_{\mu}(\sigma)$ by combining the (conformal) diffeomorphism $\sigma^{\pm}\rightarrow y^{\pm}$, where

$$y^{+}=\sigma^{+}\ ,\quad y^{-}=-\ln(e^{-\lambda\sigma^{-}}-a/\lambda)/\lambda\ , \tag{9}$$

and the Lorentz transformation $\check{e}^{\pm}_{\mu}(y)\rightarrow(1+ae^{\lambda y^{-}}/\lambda)^{\pm 1/2}\,\check{e}^{\pm}_{\mu}(y)$.
Using the fact that the energy-momentum tensor transforms covariantly under diffeomorphisms, but under Lorentz transformations it follows the transformation rules implied by the anomaly relations (5), (6), (7), one finds that the observer $\check{e}^{a}_{\mu}(y)$ sees the following $\check{T}_{--}$

$$\check{T}_{--}(y)=[\hat{T}_{--}(\sigma(y))+\Delta^{\hat{e}\rightarrow\check{e}}_{--}(\sigma(y))](d\sigma^{-}/dy^{-})^{2} \tag{10}$$

where

$$\Delta^{\hat{e}\rightarrow\check{e}}_{--}=\frac{1}{48\pi}\left\{h\left[\partial^{2}_{\sigma^{-}}\left(\ln\frac{dy^{-}}{d\sigma^{-}}\right)\right]^{2}-(2h+cq)\,\partial^{2}_{\sigma^{-}}\left(\ln\frac{dy^{-}}{d\sigma^{-}}\right)+2cq\,\partial_{\sigma^{-}}\hat{\rho}\,\partial_{\sigma^{-}}\left(\ln\frac{dy^{-}}{d\sigma^{-}}\right)\right\}\ . \tag{11}$$

Since on $\mathcal{I}^{+}_{R}$ (*i.e.* $y^{+}=\infty$) one finds that $\hat{\rho}=(1/2)\ln(dy^{-}/d\sigma^{-})$, from Eqs.(8)-(11) it follows that $\check{T}_{--}$ is h-independent on $\mathcal{I}^{+}_{R}$

$$\left[\check{T}_{--}\right]_{\mathcal{I}^{+}_{R}}=c(1-q)\frac{\lambda^{2}}{48\pi}\left[1-(1+ae^{\lambda y^{-}}/\lambda)^{-2}\right]\ . \tag{12}$$

Notice that this result reproduces the corresponding result[1, 2] for the CGHS model upon replacing $c(1-q)=N_{-}$ with N, the number of boson fields of the CGHS model. This might indicate that, in spite of the Lorentz anomaly, in the present approximation left-movers and right-movers are still decoupled[2]; in fact, such a decoupling implies that the Hawking radiation on $\mathcal{I}^{+}_{R}$ due to our black hole (formed by gravitational collapse of a left-moving shock wave) should only be sensitive to N_{-}.

The fact that the complicated structure of the anomalous Lorentz transformations of the energy momentum tensor and the anomaly relations (5), (6), (7), conspire to give the h-independent result (12) is consistent with the findings of Ref.[5], where the semiclassical analysis of the black hole radiation in the ordinary CGHS model was shown to be insensitive to the value of the coefficient of a local regularization counterterm. However, the similarities between our model and the Chiral Schwinger model (ours is essentially a gravitational version

of the Chiral Schwinger model) suggest that the parameter h should have some non-trivial physical role; for example, the Jackiw-Rajaraman parameter affects the value of the mass emergent[14] in the Chiral Schwinger model. Moreover, investigations[10] of related chiral quantum gravity theories have found that h does have a non-trivial role in the fully quantized theory; most notably, the central charge has been found to depend on it. We therefore expect that in the fully quantized theory the black hole radiation depends on h even though this is not seen in the naive semiclassical analysis. To test this expectation, in the following we shall go beyond the naive semiclassical approximation; specifically, we shall exploit the fact that it is rather simple to perform the functional integration over one of the four fields characterizing the *zweibein*, and include the corresponding quantum correction in our analysis.

A general *zweibein* e^a_μ can be written in terms of a field Υ, which we shall call *Lorentzon*, and a Lorentz gauge-fixed *zweibein* $[\hat{e}^{\pm}_\mu]_{\hat{\Upsilon}}$ (*i.e.* a *zweibein* whose orientation $\hat{\Upsilon}$ in the tangent space is prescribed, and therefore is characterized by only three fields), as follows

$$e^{\pm}_\mu = e^{\pm\Upsilon}[e^{\pm}_\mu]_{\hat{\Upsilon}} \ . \tag{13}$$

In our theory it is rather simple to integrate out[6] the field Υ; in fact, the corresponding functional integration can be cast in Gaussian form[10], and choosing a diffeomorphism and Lorentz invariant measure of the type proposed in Ref.[15], one finds that, as we shall show in detail in Ref.[13], the *Lorentzon*-integrated energy-momentum tensor satisfies the following anomaly relations[7] in the black hole background $\hat{e}^a_\mu$

$$\hat{\nabla}_\mu \hat{T}^\mu_a = 0 \ , \tag{14}$$

$$\hat{e}^a_\mu \, \hat{T}^\mu_a = [c + h + c^2q^2/(4h)]\hat{R}/(24\pi) \ , \tag{15}$$

$$\epsilon^a{}_b \, \hat{e}^b_\mu \, \hat{T}^\mu_a = 0 \ . \tag{16}$$

One can recognize that these anomaly relations become equivalent to the ones encountered[1, 2] in the ordinary CGHS model, upon replacing $c_{eff} \equiv c + h + c^2q^2/(4h)$ with N. Therefore, once the *Lorentzon* is integrated out, the semiclassical analysis of black hole radiation in our model can be performed exactly as in the ordinary CGHS model (in particular the *Lorentzon*-integrated $\hat{T}^\mu_a$ transforms covariantly under both Lorentz transformations and diffeomorphisms); this leads to the result that for the "good observer" $\check{e}^a_\mu(y)$

$$\left[\check{T}_{--}\right]_{\mathcal{I}^+_R} = \left[c + h + \frac{c^2q^2}{4h}\right] \frac{\lambda^2}{48\pi} \left[1 - (1 + a e^{\lambda y^-}/\lambda)^{-2}\right] \ . \tag{17}$$

Hence, we find again that the black hole radiation on $\mathcal{I}^+_R$ has the same functional dependence on y^- as in the semiclassical analysis[1, 2] of the CGHS model, but in the present case the overall coefficient has an h-dependence.

The specific dependence on h of the overall coefficient c_{eff} is quite interesting. In particular, c_{eff} diverges at $h = 0$, and $c_{eff} \leq N_-$ whenever $h < 0$; this appears to be in agreement

[6] The *Lorentzon* corresponds to the degree of freedom "turned on" by the anomaly. The consequences of integrating it out are therefore particularly interesting.

[7] Note that these anomaly relations receive no contribution from the *Lorentzon* measure and the Lorentz ghosts. If, instead of following the proposal of Ref.[15], one defines the measures in the ordinary way (with the real metric), the Lorentz ghosts still do not contribute to the anomaly relations, but a contribution from the *Lorentzon* measure appears. Since this contribution can be reabsorbed by increasing the value of c by 1, our analysis would not be qualitatively modified.

with the expectation that the theory be not unitary[8] (the *Lorentzon*[9] is ghost-like[9]) when $h \leq 0$. Instead, $c_{eff} \geq N_-$ (consistently with the absence[9] of ghost-like fields) whenever $h > 0$.

Our result that after integrating out the *Lorentzon* the radiation has an h-dependence, which is not present in the naive semiclassical approximation, also allows to reconcile the findings of Refs.[4, 5] with the ones of Refs.[9, 10, 14]. Notably, the $\check{T}_{--}$ in Eq.(17) takes the same value in correspondence of pairs of values of h, just like paired values of the Jackiw-Rajaraman parameter lead to the same value of the mass emergent in the Chiral Schwinger model[14].

In conclusion, we want to emphasize that in this paper we have only explored some aspects of a torsionless chiral quantum gravity theory which are relevant to the issue of the possible role of renormalization parameters (coefficients of local counterterms) in 1+1-dimensional black hole radiation; however, it is our expectation that other problems of 1+1-dimensional black hole quantum mechanics might be investigated by exploiting the rich structure of chiral quantum gravity theories. For example, it would be interesting to check whether a fully consistent analysis of the back reaction[3] (a necessary step toward a rigorous description of black hole evaporation, which has been attempted without success in the CGHS model) is possible in some (not necessarily torsionless) chiral quantum gravity theory.

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[8]Work is in progress[13] attempting to test more rigorously whether the theory with negative h is physical at least when $c_{eff} \geq 0$, *i.e.* $-c[1 + (1 - q^2)^{1/2}]/2 \leq h \leq -c[1 - (1 - q^2)^{1/2}]/2$.

[9]Note that, in the spirit of Ref.[15], we have included no contribution from the measures of the *Lorentzon*, dilaton, conformal factor, and ghosts. However, the (diffeomorphism invariant) quantization of the chiral matter fields (whose measure has to involve the physical metric[15]) still induces a kinetic term for the *Lorentzon*.

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